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Dielectric Loaded Elliptical Waveguides

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Abstract—**Wave propagation in a metallic elliptic waveguide loaded with a dielectric rod or a dielectric lining is investigated theoretically. The mode spectrum for both slow and fast wave hybrid modes is obtained by numerical solution of the characteristic equations. Correspondence is established between the modes of the loaded and unloaded elliptical waveguides. Typical field plots for $_eHE_{01}$ and $_oEH_{01}$ modes are presented. Power flow, power loss, and attenuation are obtained using a perturbation method.**

I. Introduction

THE DIELECTRIC loaded metallic elliptical waveguide has been shown to have application in acceleration devices [1], and also as a microwave heating applicator [2]. The study of metallic elliptical waveguides with two dielectric media involves the solution of an infinite determinantal equation. Veselov [3] derived the dispersion equations of all modes in this waveguide. Cutoff frequencies of some low-order modes, computed from a first-order approximation to the characteristic equation have been reported [4]–[7], while Rayevskiy *et al.* [8] have obtained the field distribution of the dominant mode. The dispersion equation of the $_eHE_{11}$ mode has been studied [9]–[11] for the special case of the phase velocity near the velocity of light because of its application in electron accelerators. The use of a second-order approximation to the dispersion equation has been reported to yield improved accuracy in the computation of cutoff frequencies and to reduce field mismatch errors [10]. The mode spectrum and propagation characteristics of this waveguide have not been reported previously.

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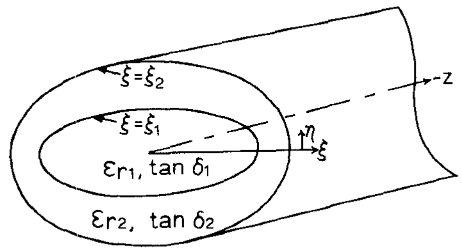

Fig. 1. Dielectric loaded elliptical waveguide and coordinate system.

In this work, the characteristic equations for fast and slow wave modes are solved numerically using accurate computer algorithms to obtain the mode spectrum. Propagation characteristics of the dominant and some higher order modes are studied theoretically, using a computational procedure similar to that employed in [12].

II. Field Components

The geometry of interest is an elliptical metallic waveguide either lined with a dielectric layer or loaded with an elliptical dielectric rod as shown in Fig. 1. The boundary layer between the two dielectric regions is an elliptical surface confocal with the metallic surface. This structure can propagate only hybrid modes which may be either slow or fast waves.

Omitting the t–z dependence, $\exp[j(\omega t-\beta z)]$, where β is the phase coefficient and ω is the angular frequency, the lossless axial field components in region i $(i=1,2)$, for even modes are

$$E_{zi}=\sum_{m=1}^{\infty} a_m^{(i)}A_m^{(i)}(\xi,q_i)se_m(\eta,q_i)$$

$$H_{zi}=\sum_{m=0}^{\infty} b_m^{(i)}B_m^{(i)}(\xi,q_i)ce_m(\eta,q_i) \quad (1)$$

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where $a_m^{(i)}$ and $b_m^{(i)}$ are arbitrary constants; $q_i=(k_i^2-\beta^2)h^2/4$; k_i is the wavenumber in region i; h is semiinterfocal distance; and $A_m^{(i)}(\xi,q_i)$, $B_m^{(i)}(\xi,q_i)$ are functions of modified Mathieu functions given by (2) and (3). For region 1, only modified Mathieu functions of the first kind are required, while functions of both kinds are required for region 2.

$$A_m^{(1)}(\xi,q_1)=Se_m(\xi,q_1)$$

$$A_m^{(2)}(\xi,q_2)=\begin{cases} Gey_m(\xi,q_2)-\dfrac{Gey_m(\xi_2,q_2)}{Se_m(\xi_2,q_2)}Se_m(\xi,q_2), & \text{for even modes}\\ Gey_m(\xi,q_2)-\dfrac{Gey_m'(\xi_2,q_2)}{Se_m'(\xi_2,q_2)}Se_m(\xi,q_2), & \text{for odd modes}\end{cases} \tag{2}$$

$$B_m^{(1)}(\xi,q_1)=Ce_m(\xi,q_1)$$

$$B_m^{(2)}(\xi,q_2)=\begin{cases} Fey_m(\xi,q_2)-\dfrac{Fey_m'(\xi_2,q_2)}{Ce_m'(\xi_2,q_2)}Ce_m(\xi,q_2), & \text{for even modes}\\ Fey_m(\xi,q_2)-\dfrac{Fey_m(\xi_2,q_2)}{Ce_m(\xi_2,q_2)}Ce_m(\xi,q_2), & \text{for odd modes.}\end{cases} \tag{3}$$

Note: When $q_2<0$, functions Gey_m, Gey_m', Fey_m, and Fey_m' are replaced by Gek_m, Gek_m', Fek_m, and Fek_m', respectively.

Rayevskiy *et al.* [8] have studied the field distribution of the dominant ${}_eHE_{11}$ mode in an elliptical waveguide loaded with a dielectric rod. Radial variations of field components of the ${}_eHE_{01}$ and ${}_oEH_{01}$ modes in a dielectric loaded elliptical waveguide are illustrated in Figs. 2 and 3. The axial magnetic field of the ${}_oEH_{01}$ mode is small relative to the other field components. It vanishes as the ellipse degenerates to a circle and the mode becomes E_{01} in the circular guide. A similar behavior exists for the axial electric field for the ${}_eHE_{01}$ mode which degenerates to the H_{01} mode in the circular case. The ${}_oEH_{11}$ mode is found to exhibit a high concentration of electric field strength near the center of a metallic elliptical waveguide with a dielectric load, and thus has possible application as a microwave heating applicator.

Expressions for the transverse field components are given by [12, eq. (4)], along with the method for obtaining the field components for odd modes.

III. Mode Spectrum

The mode spectrum is obtained by solving the characteristic equations given in [12] except that the functions $A_m^{(i)}(\xi,q_i)$, $B_m^{(i)}(\xi,q_i)$ are given by (2) and (3). The first ten modes in a metallic elliptical waveguide loaded with a

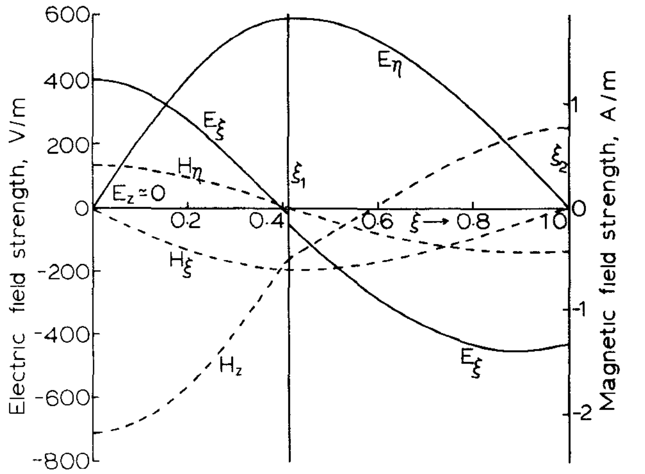


Fig. 2. Radial dependence of electric and magnetic fields of the ${}_eHE_{01}$ mode for $\varepsilon_{r1}=2.26$, $\varepsilon_{r2}=1.0$, $\xi_1=0.41$, $\xi_2=1.0$, $\beta/k_0=0.4$, $\eta=0.23\pi$, $(h\cosh\xi_2)/\lambda_0=0.674$.

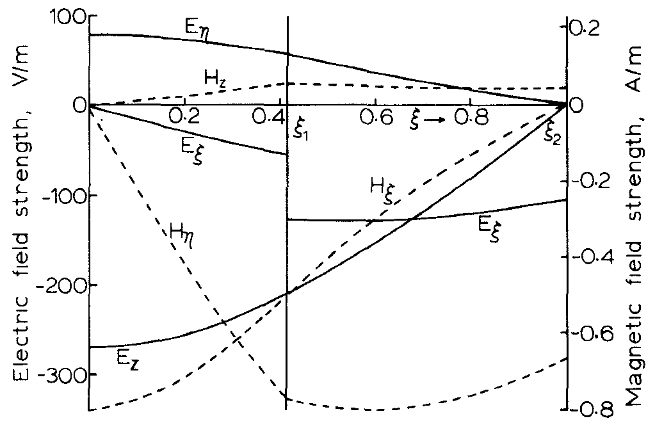


Fig. 3. Radial dependence of electric and magnetic fields of the ${}_oEH_{01}$ mode for $\varepsilon_{r1}=2.26$, $\varepsilon_{r2}=1.0$, $\xi_1=0.41$, $\xi_2=1.0$, $\beta/k_0=0.4$, $(h\cosh\xi_2)/\lambda_0=0.359$, $\eta=0.23\pi$.

dielectric rod are shown in Fig. 4. The mode designations are consistent with the definitions of odd and even hybrid modes adopted for elliptical dielectric waveguides [13]. The correspondence between hybrid modes in a two-media waveguide and previously accepted designations of TE and TM modes in a hollow metallic waveguide are shown in Table I.

Interchanging prescripts e and o to obtain the correspondence between EH_{mn} and TM_{mn} modes is required because of the different definitions of even and odd modes which have been adopted for the two cases [14].

When the elliptical cross section degenerates to the circular case, both odd and even HE_{mn} (EH_{mn}) modes for $m\geqslant 1$ smoothly degenerate to HE_{mn} (EH_{mn}) modes, while ${}_eHE_{0n}$ and ${}_oEH_{0n}$ modes degenerate to H_{0n} and E_{0n} modes. It is to be noted that ${}_oHE_{0n}$ and ${}_eEH_{0n}$ modes do not exist.

It has been reported [12] that the characteristic equations do not yield simple closed form solution under cutoff condition. However, cutoff frequencies are obtained from numerical solution of the characteristic equa-

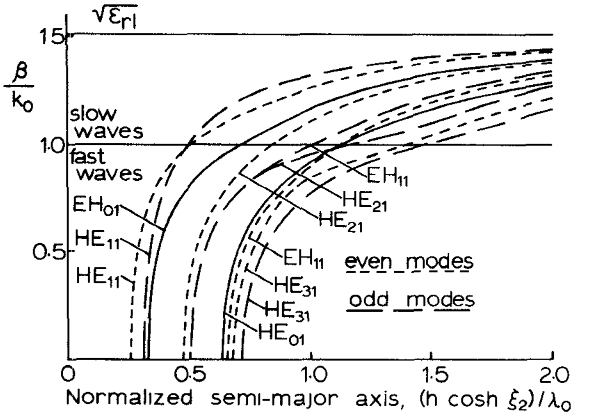


Fig. 4. Mode spectrum of the dielectric rod loaded elliptical waveguide for $\varepsilon_{r1}=2.26$, $\varepsilon_{r2}=1.0$, $\xi_1=0.41$, $\xi_2=1.0$.

TABLE I
MODE CORRESPONDENCE OF ELLIPTICAL GUIDES

Two-Media Waveguide	Hollow Metallic Waveguide	
${}_e\mathrm{HE}_{mn}$	${}_e\mathrm{TE}_{mn}$	$m \geq 0$
${}_o\mathrm{HE}_{mn}$	${}_o\mathrm{TE}_{mn}$	$m \geq 1$
${}_e\mathrm{EH}_{mn}$	${}_o\mathrm{TM}_{mn}$	$m \geq 1$
${}_o\mathrm{EH}_{mn}$	${}_e\mathrm{TM}_{mn}$	$m \geq 0$

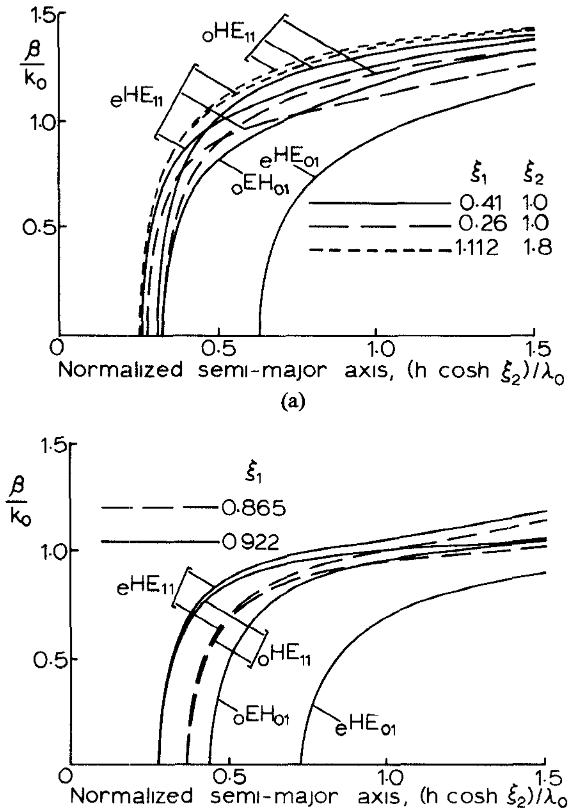
tions for $\beta/k_0 \to 0$. Extensive results have been reported for some lower order modes [4]–[7]. The mode separation between the first two modes is found to be greater for a waveguide with a homogeneous medium than for the dielectric-loaded case. Cutoff frequencies computed in this work were found to agree within 1 percent of those obtained by Kovshov *et al.* [4].

IV. PROPAGATION CHARACTERISTICS

Expressions for power flow, energy storage, dielectric loss, and group velocity are identical to the corresponding ones given in [12]. However the expression for conductor loss in the case of even modes is obtained from [12, eq. (15)] by replacing the parameters listed in Appendix A.

A. Phase Characteristics

The variation of normalized phase coefficient with normalized semimajor axis for ${}_e\mathrm{HE}_{11}$, ${}_o\mathrm{HE}_{11}$, ${}_e\mathrm{HE}_{01}$, and ${}_o\mathrm{EH}_{01}$ modes are illustrated in Figs. 5(a) and (b). Near cutoff, the phase coefficient of the ${}_o\mathrm{HE}_{11}$ mode in the rod loaded guide is lower than that of the ${}_e\mathrm{HE}_{11}$ mode, whereas in the slow wave region it is greater. Also, slow wave propagation is similar to that along the dielectric rod waveguide. The difference in phase coefficient of the ${}_e\mathrm{HE}_{11}$ and ${}_o\mathrm{HE}_{11}$ modes increases with eccentricity. Increasing the dielectric constant or the filling factor reduces the cutoff frequencies of all modes. In the case of the dielectric lined waveguide, the phase coefficient of the HE_{11} modes resemble that of the empty waveguide near cutoff. With increasing dielectric thickness the characteristics show an increasing deviation from those of the empty guide and are slow-varying functions near $\beta/k_0 = 1$ as illustrated in Fig. 5(b).



(b)

Fig. 5. (a) Phase characteristics of dielectric rod loaded elliptical waveguide for $\varepsilon_{r1}=2.26$, $\varepsilon_{r2}=1.0$. (b) Phase characteristics of dielectric lined elliptical waveguide for $\varepsilon_{r1}=1.0$, $\varepsilon_{r2}=2.26$, $\xi_2=1.0$.

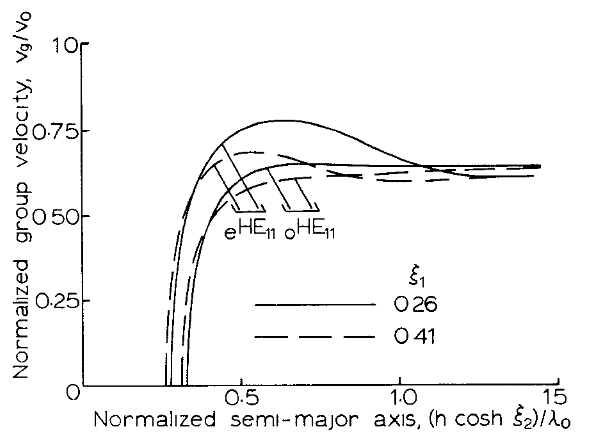
B. Group Velocity

The group velocity derived from energy considerations in [12] is used to obtain the normalized group velocity characteristics which are illustrated in Figs. 6(a) and (b) for the dielectric rod loaded guide and the lined guide, respectively. It is seen that the ${}_o\mathrm{HE}_{11}$ mode on the shielded rod has a flat group velocity characteristic.

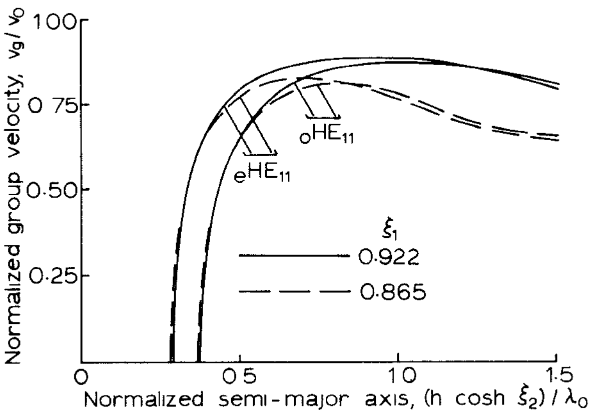
C. Attenuation characteristics

Attenuation is obtained from conductor and dielectric losses per unit length and power flow, by the perturbation technique described in [12]. Typical attenuation characteristics of ${}_e\mathrm{HE}_{11}$ and ${}_o\mathrm{HE}_{11}$ modes are shown in Figs. 7(a) and (b).

The attenuation characteristics of the lined waveguide are found to be flat over a wider band than those of rod loaded guides as shown in Figs. 7(a) and (b). The flat characteristic can be produced by a suitable choice of dielectric constant and fill factor, although the attenuation is higher than that of the empty guide by an order of magnitude. The ${}_e\mathrm{HE}_{11}$ mode in the dielectric rod loaded guide exhibits low attenuation when the eccentricities are high and the rod dimensions are relatively small compared to the dimensions of the metal. Under these conditions the field is essentially that of a shielded surface wave. Computed attenuation coefficients for a dielectric loaded guide of $\xi_1=0.15$, $\xi_2=1.0$, $h\cosh\xi_2=1.75$ cm, $\varepsilon_{r1}=$

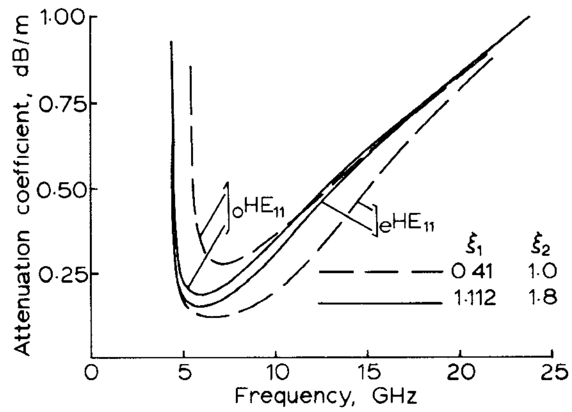


(a)

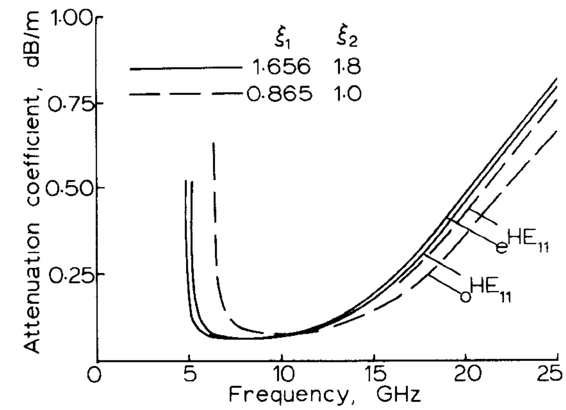


(b)

Fig. 6. (a) Group velocity characteristics of dielectric rod loaded elliptical waveguide for $\varepsilon_{r1}=2.26$, $\varepsilon_{r2}=1.0$, $\xi_2=1.0$. (b) Group velocity characteristics of dielectric lined elliptical waveguide for $\varepsilon_{r1}=1.0$, $\varepsilon_{r2}=2.26$, $\xi_2=1.0$.



(a)



(b)

Fig. 7. (a) Attenuation characteristics of dielectric rod loaded elliptical waveguide for $\varepsilon_{r1}=2.26$, $\varepsilon_{r2}=1.0$, $\tan\delta_1=0.0003$, $\tan\delta_2=0$, $\sigma=5.8\times10^7$ S/m, $h\cosh\xi_2=1.75$ cm. (b) Attenuation characteristics of dielectric lined elliptical waveguide for $\varepsilon_{r1}=1.0$, $\varepsilon_{r2}=2.26$, $\tan\delta_1=0$, $\tan\delta_2=0.0003$, $\sigma=5.8\cdot10^7$ S/m, $h\cosh\xi_2=1.75$ cm.

2.26, $\varepsilon_{r2}=1.0$, $\sigma=5.8\cdot10^7$ S/m, and $\tan\delta=3\times10^{-4}$ are 0.08 dB/m at 8.6 GHz and 0.086 at 10 GHz for the ${}_e\mathrm{HE}_{11}$ mode. The corresponding results for the ${}_o\mathrm{HE}_{11}$ mode are 0.278 dB/m and 0.312 dB/m. The attenuation coefficients for the same waveguide without loading were computed from Kretzschmar's expressions [15] and are 0.037 dB/m and 0.034 dB/m for ${}_e\mathrm{TE}_{11}$ mode and 0.056 dB/m and 0.042 dB/m for ${}_o\mathrm{TE}_{11}$ mode at 8.6 GHz and 10 GHz, respectively.

V. Conclusion

This work has presented the mode spectrum of the elliptical metallic waveguide loaded with a confocal dielectric rod. As a result the mode correspondence between this waveguide and an empty waveguide are established. Computed results for propagation characteristics show that a metallic waveguide with two dielectric media can have low dispersion. Surface-wave transmission with very low attenuation is possible along a dielectric rod waveguide with a metallic shield. These guides have possible application in electron accelerators, as microwave heating applicators and for shielded surface wave transmission.

Appendix A

Parameters in the expression for conductor loss

Surface-wave transmission line		Dielectric loaded guide
$a_m^{(1)}$	becomes	$a_m^{(2)}$
$a_n^{(1)}$	"	$a_n^{(2)}$
$b_m^{(1)}$	"	$b_m^{(2)}$
$b_n^{(1)}$	"	$b_n^{(2)}$
$A_m^{(1)\prime}(\xi_0,q_1)$	"	$A_m^{(2)\prime}(\xi_2,q_2)$
$A_n^{(1)\prime}(\xi_0,q_1)$	"	$A_n^{(2)\prime}(\xi_2,q_2)$
$B_m^{(1)}(\xi_0,q_1)$	"	$B_m^{(2)}(\xi_2,q_2)$
$B_n^{(1)}(\xi_0,q_1)$	"	$B_n^{(2)}(\xi_2,q_2)$
$\dfrac{e=1}{\cosh\xi_0}$	"	$\dfrac{e=1}{\cosh\xi_2}$
h_1	"	h_2
q_1	"	q_2
ε_1	"	ε_2
ξ_0	"	ξ_2

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The Elliptical Surface Wave Transmission Line

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***Abstract*—Electromagnetic wave propagation on an elliptical cross-sectional surface-wave transmission line is investigated theoretically. Characteristic equations for odd and even hybrid modes are derived and solved numerically. Expressions are obtained for power flow, energy storage and power loss using a perturbation method. Numerical results on propagation characteristics of three lower order modes are presented. The $_oHE_{11}$ mode is shown to have low attenuation particularly at high eccentricities. The propagation characteristics of lines of high eccentricities are found to be slowly varying functions of dimensions.**

I. Introduction

SINGLE WIRE transmission lines and dielectric coated conductors of circular cross sections have been studied extensively [1]–[5]. King and Wiltse [4] have shown that the circular Goubau line has application in millimeter wave propagation because of low attenuation.

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Cutoff wave numbers of several low-order modes in Goubau lines have been reported recently [5]. The even dominant mode in elliptical dielectric rod and tube waveguides has been shown to have lower attenuation than the corresponding mode in circular dielectric waveguides. Also, the attenuation is a slowly varying function of dimensions in the elliptical case, resulting in greater dimensional tolerances [6]–[9]. It follows that Goubau lines of elliptical cross sections should exhibit improved propagation characteristics over circular cases.

Karbowiak's [10] analysis of the elliptical Goubau line has very limited applications since he considered only one term in the infinite series for field expressions. Roumeliotis *et al.* [11] have obtained wavenumbers of certain modes in the above waveguide for small eccentricities only. Propagation characteristics of elliptical Goubau lines have not been reported.

In this work, the elliptical Goubau line is studied theoretically using the perturbation method and numerical results for propagation characteristics are presented.

II. Field Components

The elliptical Goubau line consists of an elliptical cylindrical conductor coated with a confocal dielectric layer as shown in Fig. 1. Though a constant dielectric thickness

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